AMC WARM-UP PAPER
SENIOR PAPER 7
SOLUTIONS

1. $3^{x+1} = 81 \rightarrow 3^{x+1} = 3^4 \rightarrow x = 3$,

hence (C).

2.

$$\begin{aligned} \frac{m}{m-n} + \frac{n}{n-m} &= \frac{m}{m-n} - \frac{n}{m-n} \\ &= \frac{m-n}{m-n} \\ &= 1, \end{aligned}$$

hence (D).

3. If each of the 12 houses gets 4 letters then there are 9 letters left. Giving one more letter to 8 houses (including George's) means that George must also get the last letter, so he must get at least 6 letters,

hence (D).

4. From the data,

$$\begin{aligned} f(x) &= \frac{1}{1+x} \\ f(f(x)) &= \frac{1}{1+\frac{1}{1+x}} \\ &= \frac{x+1}{x+1+1} \\ &= \frac{x+1}{x+2}, \end{aligned}$$

hence (B).

5. **Alternative 1**

Now $70 = 2 \times 5 \times 7$, so we only have to look for two factors differing by 2 (as the difference in the capacity of the cases is 2 kg), and in this instance it must be 5 and 7, so the capacity of the standard case is 5 kg,

hence (B).

Note: in this particular example we do not need to know that 4 less cases are used.

Alternative 2

Let the capacity of the smaller cases be x, then the capacity of the larger is $x + 2$ and

$$\begin{aligned} \frac{70}{x} - \frac{70}{x+2} &= 4 \\ 70(x+2) - 70x &= 4x^2 + 8x \\ x^2 + 2x - 35 &= 0 \\ (x+7)(x-5) &= 0 \\ x &= 5 \text{ (neglecting the negative root),} \end{aligned}$$

hence (B).

6. Landing within 1 m from the hole is landing within a circle radius 1 m, i.e. with area $\pi \times 1^2 = \pi$.

The area of the green is $\pi \times 12^2 = 144\pi$.

Thus the probability of the ball landing within 1 m from the hole is

$$\frac{\pi}{144\pi} = \frac{1}{144},$$

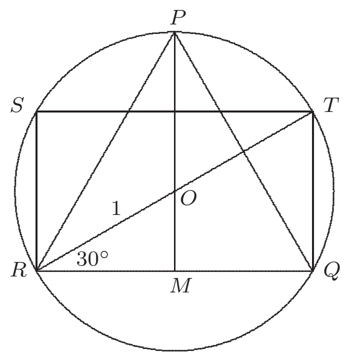
hence (E).

7. The average of the n numbers is k so their sum is kn. When x is added, there are $n + 1$ numbers and their average becomes $k + 1$. So

$$\begin{aligned} \frac{kn + x}{n+1} &= k + 1 \\ kn + x &= (n+1)(k+1) \\ &= kn + k + n + 1 \\ \text{thus} \qquad x &= k + n + 1\,, \end{aligned}$$

hence (A).

8. Join RT. This passes through the centre of the circle O as it subtends a right angle at the circumference.



Thus $\angle TRQ = 30°$. So, from the right triangle TRQ $TQ = 2\sin 30° = 2 \times \frac{1}{2} = 1$. Also, $RQ = 2\cos 30° = \sqrt{3}$.

Thus the area of the rectangle $RSTQ = 1 \times \sqrt{3} = \sqrt{3}$,

hence (E).

9. Alternative 1

The total number of ways of choosing 3 numbers from 10 is $\binom{10}{3} = 120$.

There are 8 triples containing the pair 1,2;
8 containing the pair 2,3;
⋮
8 containing the pair 9,10,
i.e 72 such triples.

However we have counted twice the triples 123, 234, $\cdots$, 8910, so only 64 triples are excluded. This leaves $120 - 64 = 56$ triples available,

hence (C).

Alternative 2

Suppose $1 \le a < b < c \le 10$ are such that no two of a, b and c are consecutive.

Let $d = b - 1$ and $e = c - 2$. Then $1 \le a < d < e \le 8$.

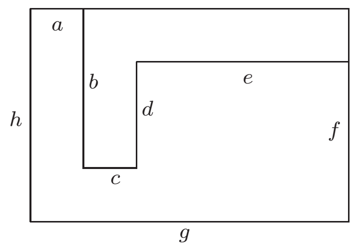
Conversely, suppose $1 \le a < d < e \le 8$.

Let $b = d + 1$ and $c = e + 2$. Then $1 \le a < b < c \le 10$ and no two of a, b and c are consecutive.

Thus the number of suitable (a, b, c) is equal to the number of suitable (a, d, e), which is $\binom{8}{3} = 56$,

hence (C).

10. Label the sides of the octagon as shown.



Label the sides of the octagon as shown.

If $g = 7$, then $h = 8$ and $\{a, c, e\} = \{1, 2, 4\}$.

Since $b + f = d + h$, then $d = 3$.

To maximise the area of the hexagon, we must have $a = 1$, $c = 4$, $e = 2$, $b = 6$ and $f = 5$, giving an area of 30.

If $g = 8$, then $h = 7$ and $\{a, c, e\} = \{1, 2, 5\}$ or $\{1, 3, 4\}$.

Since $b + f = d + h, d = 4$ and $\{h, f\} = \{4, 6\}$.

To maximise the area of the hexagon, we must have $a = 1$, $c = 5$, $e = 2$, $b = 6$ and $f = 4$, giving an area of 36,

hence (E).