

雪兰莪暨吉隆坡福建会馆  
新 纪 元 学 院

联合主办

**ANJURAN BERSAMA  
PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR  
&  
KOLEJ NEW ERA**

**第二十九届（2014 年度）  
雪隆中学华罗庚杯数学比赛**

**PERTANDINGAN MATEMATIK PIALA HUA LO-GENG  
ANTARA SEKOLAH-SEKOLAH MENENGAH  
DI NEGERI SELANGOR DAN KUALA LUMPUR  
YANG KE-29 (2014)**

**~~ 高中组 ~~  
BAHAGIAN MENENGAH TINGGI**

日期 : 2014 年 8 月 10 日 (星期日)  
Tarikh : 10 Ogos 2014 (Hari Ahad)

时间 : 10:00 → 12:00 (两小时)  
Masa : 10:00 → 12:00 (2 jam)

地点 : 新纪元学院 UG 活动中心

Tempat : UG Hall Kolej New Era  
Blok C, Lot 5, Seksyen 10, Jalan Bukit,  
43000 Kajang, Selangor.

**\*\*\* 说明 \*\*\***

1. 不准使用计算机。
2. 不必使用对数表。
3. 对一题得4分，错一题倒扣1分。
4. 答案E: 若是“以上皆非”或“不能确定”，一律以“\*\*\*”代替之。

**\*\*\* INSTRUCTIONS \*\*\***

1. Calculators not allowed.
2. Logarithm table is not to be used.
3. 4 marks will be awarded for each correct answer and 1 mark will be deducted for each wrong answer.
4. (E) \*\*\* indicates “none of the above”.

- 
1. 知  $S = \left(1 + \frac{2}{3}\right)\left(1 + \frac{2}{5}\right)\left(1 + \frac{2}{7}\right)\dots\left(1 + \frac{2}{1355}\right)\left(1 + \frac{2}{1357}\right)$  是整数，求  $S$  的各位数字之和。

Given that  $S = \left(1 + \frac{2}{3}\right)\left(1 + \frac{2}{5}\right)\left(1 + \frac{2}{7}\right)\dots\left(1 + \frac{2}{1355}\right)\left(1 + \frac{2}{1357}\right)$  is an integer, find the sum of all the digits of  $S$ .

- A 18                      B 16                      C 14                      D 12                      E \*\*\*

2. 在  $\triangle ABC$  中， $\sin A = \frac{15}{17}$ ， $\cos B = \frac{5}{13}$ ，求  $\cos C$  的值。

In  $\triangle ABC$ ,  $\sin A = \frac{15}{17}$ ,  $\cos B = \frac{5}{13}$ , find the value of  $\cos C$ .

- A  $\frac{140}{221}$                       B  $\frac{220}{221}$                       C  $-\frac{220}{221}$                       D  $-\frac{140}{221}$                       E \*\*\*

3. 已知  $x = 7^{2013}11^{2014}13^{2015}$ ，求  $x$  的最后两位数。

Given that  $x = 7^{2013}11^{2014}13^{2015}$ , find the last two digits of  $x$ .

- A 49                      B 59                      C 69                      D 79                      E \*\*\*

4. 已知  $\alpha$  是实数， $a$ 、 $b$  是方程式  $x^2 + 2x \cos \alpha + 1 = 0$  的两个根， $c$ 、 $d$  是方程式  $x^2 + 2x \cos \alpha + 1 = 0$  的两个根。求  $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}$  的值。

Given that  $\alpha$  is a real number,  $a$ ,  $b$  are the two roots of the equation  $x^2 + 2x \cos \alpha + 1 = 0$ ,  $c$ ,  $d$  are the two roots of the equation  $x^2 + 2x \cos \alpha + 1 = 0$ , find the value of  $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}$ .

- A -4                      B -2                      C 0                      D 2                      E \*\*\*

5. 已知  $(19.8)^a = (0.00000198)^b = 100$ ，求  $\frac{1}{a} - \frac{1}{b}$  的值。

Given that  $(19.8)^a = (0.00000198)^b = 100$ , find the value of  $\frac{1}{a} - \frac{1}{b}$ .

- A 3                      B 4                      C 6                      D 7                      E \*\*\*

6. 若  $a = \frac{1 + \sqrt{2017}}{2}$ ,  $c = a^3 - 505a - 505$ , 求  $c$  的值。

If  $a = \frac{1 + \sqrt{2017}}{2}$ ,  $c = a^3 - 505a - 505$ , find the value of  $c$ .

**A** -1                      **B** -2                      **C** 1                      **D** 2                      **E** \*\*\*

7. 已知  $\sin \theta - \cos \theta = \frac{\sqrt{30} - \sqrt{6}}{4}$ , 若  $x = (\sin^3 \theta - \cos^3 \theta)^2$ , 求  $32x$  的值。

Given that  $\sin \theta - \cos \theta = \frac{\sqrt{30} - \sqrt{6}}{4}$ . If  $x = (\sin^3 \theta - \cos^3 \theta)^2$ , find the value of  $32x$ .

**A** 25                      **B** 27                      **C** 33                      **D** 35                      **E** \*\*\*

8. 求  $p(x) = (x+2)^{101} + (x+3)^{200}$  除以  $x^2 + 5x + 6$  的余式。

Find the remainder when  $p(x) = (x+2)^{101} + (x+3)^{200}$  is divided by  $x^2 + 5x + 6$ .

**A** 0                      **B** 1                      **C**  $2x - 3$                       **D**  $2x + 5$                       **E** \*\*\*

9. 求函数  $f(x) = x^2 + 2\sqrt{4-x^2}$ ,  $-2 \leq x \leq 2$  的值域。

Find the range of the function  $f(x) = x^2 + 2\sqrt{4-x^2}$ ,  $-2 \leq x \leq 2$ .

**A**  $4 \leq f(x) \leq 1 + 2\sqrt{3}$                       **B**  $4 \leq f(x) \leq 5$                       **C**  $3 \leq f(x) \leq 5$   
**D**  $3 \leq f(x) \leq 1 + 2\sqrt{3}$                       **E** \*\*\*

10. 求  $\frac{3}{\sin 50^\circ} + 12 \sin 10^\circ$ 。

Find  $\frac{3}{\sin 50^\circ} + 12 \sin 10^\circ$ .

**A** 3                      **B** -3                      **C** 6                      **D** -6                      **E** \*\*\*

11. 将 2217 本书装入  $N$  个箱子中使得每个箱子都装有相同数量的书, 结果会多出 7 本书。问  $N$  的可能值有多少个?

When 2217 books are packed into  $N$  boxes such that every box contains the same number of books, there will be 7 books remain. How many possible values can  $N$  have?

**A** 8                      **B** 12                      **C** 13                      **D** 15                      **E** \*\*\*

12. 已知  $x$ 、 $y$  是实数且满足  $x^2 + y^2 = 8x - 6y + 144$ 。求  $x^2 + y^2$  的最小值。

Given that  $x$ ,  $y$  are real numbers satisfying  $x^2 + y^2 = 8x - 6y + 144$ , find the smallest value of  $x^2 + y^2$ .

**A** 64                      **B** 68                      **C** 70                      **D** 72                      **E** \*\*\*

13. 已知直角  $\triangle ABC$  的面积为  $84\text{cm}^2$ ,  $\angle A = 90^\circ$ ,  $BC = 25\text{cm}$ , 求  $\triangle ABC$  的内切圆的半径。

Given that the area of the right-angled  $\triangle ABC$  is  $84\text{cm}^2$ ,  $\angle A = 90^\circ$ ,  $BC = 25\text{cm}$ . Find the radius of the inscribed circle of  $\triangle ABC$ .

**A** 2cm                      **B** 3cm                      **C** 4cm                      **D** 5cm                      **E** \*\*\*

14. 有多少个正整数  $x$  满足不等式  $\left| 4 + \log_x \frac{1}{5} \right| < \frac{11}{3}$ ?

How many positive integers  $x$  satisfy the inequality  $\left| 4 + \log_x \frac{1}{5} \right| < \frac{11}{3}$ ?

- A 120                      B 121                      C 122                      D 123                      E \*\*\*

15. 求介于 1000 与 3000 之间能被 14 或 18 (或两者) 整除的整数的个数。

Find the number of integers between 1000 and 3000 that are divisible by 14 or 18 (or both).

- A 253                      B 254                      C 237                      D 238                      E \*\*\*

16. 一位老师要学生求出 3375, 7425 及  $N$  三个自然数的最小公倍数。其中一位学生误将第一个数 3375 看成 3675, 但他求出的结果却一样。若  $M$  是  $N$  的最小可能值,  $D$  为 3375, 7425 及  $M$  的最大公因数, 求  $M + D$  的值。

A teacher wants his students to find the least common multiple of the three natural numbers 3375, 7425 and  $N$ . One of the students misread the first number 3375 as 3675, but the result he obtained is the same. If  $M$  is the smallest possible value of  $N$ ,  $D$  is the greatest common factor of 3375, 7425 and  $M$ , find the value of  $M + D$ .

- A 6150                      B 6175                      C 2205                      D 2250                      E \*\*\*

17. 若  $A$ 、 $B$  是平面上相距 50cm 的两点, 平面上有多少条直线  $L$  使得  $A$  到  $L$  最近的距离为 14cm,  $B$  到  $L$  最近的距离为 26cm?

If  $A, B$  are two points on the plane with distance 50cm apart, how many lines  $L$  on the plane are such that the minimal distance from  $A$  to  $L$  is 14cm, and the minimal distance from  $B$  to  $L$  is 26cm?

- A 1                          B 3                          C 4                          D 无限多                      E \*\*\*  
ininitely many

18. 如图 1 所示,  $ABCD$  是正方形。  $R$  在  $DC$  的延长线上,  $AR$  与  $BD$  相交于  $P$ , 与  $BC$  相交于  $Q$ 。 已知  $AP = 28\text{cm}$ ,  $PQ = 16\text{cm}$ , 求  $QR$  的长。

As shown in the Figure 1,  $ABCD$  is a square.  $R$  is a point on the extension of the line  $DC$ .  $AR$  intersects  $BD$  at  $P$ , and intersects  $BC$  at  $Q$ . Given that  $AP = 28\text{cm}$ ,  $PQ = 16\text{cm}$ , find the length of  $QR$ .

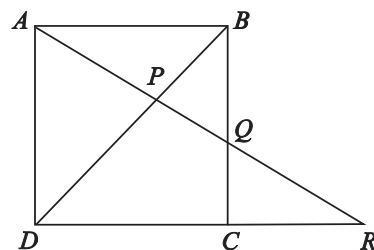


图 1  
Figure 1

- A 28cm                      B 30cm                      C 32cm  
D 33cm                      E \*\*\*

19. 求  $\frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} + \dots + \frac{50}{48!+49!+50!}$  的值。

Find the value of  $\frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} + \dots + \frac{50}{48!+49!+50!}$ .

- A  $1 - \frac{1}{50!}$                       B  $\frac{1}{2!} - \frac{1}{50!}$                       C  $1 - \frac{1}{49!}$                       D  $\frac{1}{2!} - \frac{1}{49!}$                       E \*\*\*

20. 如图 2 所示,  $AB$  是半圆的直径,  $C, D$  是半圆上两点。已知  $AB = 8, AC = 2, CD = 2$ , 求  $BD$  的长。

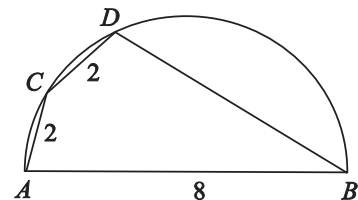


图 2  
Figure 2

As shown in the Figure 2,  $AB$  is the diameter of the semi-circle,  $C, D$  are two points on the semi-circle. Given that  $AB = 8, AC = 2, CD = 2$ , find the length of  $BD$ .

- A 7                      B 6                      C  $2\sqrt{10}$   
D  $5\sqrt{2}$                 E \*\*\*
21. 求所有由 1、2、3、4、5、6 组成可重复数字的三位数之和。  
Find the sum of all the three-digit numbers formed using the digits 1, 2, 3, 4, 5, 6 (where repetition is allowed).  
A 46620                B 69300                C 83160                D 83916                E \*\*\*
22. 在坐标平面上, 曲线  $y = x^3 - 4x + 2$  与曲线  $y = \frac{8}{x^2}$  相交于多少个不同的点?  
In the coordinate plane, the curve  $y = x^3 - 4x + 2$  and the curve  $y = \frac{8}{x^2}$  intersect at how many distinct points?  
A 0                      B 2                      C 3                      D 5                      E \*\*\*
23. 由五个 0 及五个 1 所组成的二元序列中, 有多少个没有三个连在一起的 0?  
(注: 1010101010, 0011001110 等都是这样的序列, 而 0001110011 则不是, 因其有三个连在一起的 0。)  
Among the binary sequence formed by five 0's and five 1's, how many of them do not have three consecutive 0's?  
(Note: 1010101010, 0011001110, etc, are such sequences, but 0001110011 is not, since it has three consecutive 0's.)  
A 252                      B 132                      C 126                      D 51                      E \*\*\*
24. 设  $x$  为有 1000 位数字的数 456745674567...4567。若  $x$  的前面  $m$  位数字与后面  $n$  位数字被去掉后, 剩下的数其各位数字之和为 4567, 求  $m+n$  的值。  
Let  $x$  be the number 456745674567...4567 that has 1000 digits. If the first  $m$  digits and the last  $n$  digits of  $x$  are crossed out, the sum of the remaining digits is 4567, find the value of  $m+n$ .  
A 168                      B 169                      C 170                      D 171                      E \*\*\*
25. 若  $\omega \neq 1$  是一个满足  $\omega^5 = 1$  的复数, 求  $\sum_{n=1}^{39} \omega^{3n}$  的值。  
If  $\omega \neq 1$  is a complex number such that  $\omega^5 = 1$ , find the value of  $\sum_{n=1}^{39} \omega^{3n}$ .  
A -1                      B 0                      C 1                      D 39                      E \*\*\*

26. 华罗庚杯数学比赛中共有 35 道选择题，每一题答对得 4 分，答错扣 1 分，不答得 0 分。则每一位参赛者的得分可有多少个不同的可能值？

In the Hua Lo-Geng Cup mathematics competition, there are altogether 35 multiple choice questions. For each question, 4 marks will be awarded if answered correctly, 1 mark will be deducted if answered incorrectly, and no mark will be given if not answered. Then for the marks obtained by a candidate, how many different possible values can there be?

- A 165                      B 170                      C 172                      D 175                      E \*\*\*

27. 方程式  $\left| \frac{2|x-1|+3}{3|x+2|-10} \right| = 1$  有多少个不同的解？

How many different solutions does the equation  $\left| \frac{2|x-1|+3}{3|x+2|-10} \right| = 1$  have?

- A 2                              B 3                              C 4                              D 6                              E \*\*\*

28. 求  ${}^{999}C_0 - {}^{999}C_2 + {}^{999}C_4 - {}^{999}C_6 + \dots + {}^{999}C_{996} - {}^{999}C_{998}$  的值。

Find the value of  ${}^{999}C_0 - {}^{999}C_2 + {}^{999}C_4 - {}^{999}C_6 + \dots + {}^{999}C_{996} - {}^{999}C_{998}$ .

- A  $-2^{500}$                       B  $-2^{499}$                       C  $2^{500}$                       D  $2^{499}$                       E \*\*\*

29. 求  $\sqrt{(14+6\sqrt{5})^3} + \sqrt{(14-6\sqrt{5})^3}$ 。

Find  $\sqrt{(14+6\sqrt{5})^3} + \sqrt{(14-6\sqrt{5})^3}$ .

- A 144                              B 72                              C  $64\sqrt{5}$                       D  $32\sqrt{5}$                       E \*\*\*

30. 已知  $(a-b)(b-c)(c-a) = 3$ ,

$$S = (a-b)(99-c)(999-2c) + (b-c)(99-a)(999-2a) + (c-a)(99-b)(999-2b),$$

求  $S$  的值。

Given that  $(a-b)(b-c)(c-a) = 3$ , and

$$S = (a-b)(99-c)(999-2c) + (b-c)(99-a)(999-2a) + (c-a)(99-b)(999-2b),$$

find the value of  $S$ .

- A 6                                      B 3                                      C -3                                      D -6                                      E \*\*\*

31. 已知曲线  $y = \frac{3x-1}{x+1}$  在直角坐标平面上经直线  $y = -x$  反射后的像为  $y = f(x)$ , 求  $f(x)$ 。

Given that under the reflection about the line  $y = -x$  in the Cartesian coordinate plane, the image of the curve

$$y = \frac{3x-1}{x+1} \text{ is } y = f(x), \text{ find } f(x).$$

- A  $\frac{3x+1}{x-1}$                       B  $\frac{x-1}{x+3}$                       C  $-\frac{3x+1}{x-1}$                       D  $-\frac{x-1}{x+3}$                       E \*\*\*

32. 如图 3 所示,  $D$  及  $E$  分别是  $\triangle ABC$  中  $AB$  及  $AC$  边上的点,  $AB = 5DB$ ,  $AC = 3AE$ . 若四边形  $BCED$  的面积为  $66\text{cm}^2$ , 求  $\triangle ADE$  的面积。

As shown in the Figure 3,  $D$  and  $E$  are respectively points on the sides  $AB$  and  $AC$  of  $\triangle ABC$ ,  $AB = 5DB$ ,  $AC = 3AE$ . If the area of the quadrilateral  $BCED$  is  $66\text{cm}^2$ , find the area of  $\triangle ADE$ .

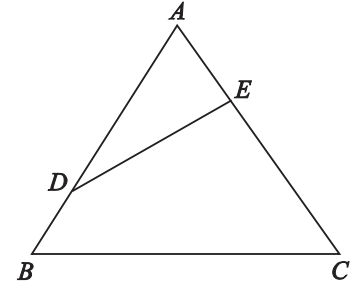


图 3  
Figure 3

- A  $22\text{cm}^2$             B  $24\text{cm}^2$             C  $27\text{cm}^2$   
D  $33\text{cm}^2$             E \*\*\*

33. 若  $n$  是正整数使得  $\sqrt{n-150} + \sqrt{n+150}$  为有理数,  $N$  为  $n$  的最大可能值, 求  $N$  除以 100 的余数。

If  $n$  is a positive integer such that  $\sqrt{n-150} + \sqrt{n+150}$  is a rational number and  $N$  is the largest possible value of  $n$ , find the remainder when  $N$  is divided by 100.

- A 50            B 26            C 13            D 1            E \*\*\*

34. 求 10800 的所有正因数和。

Find the sum of all the positive factors of 10800.

- A 18600            B 32400            C 38400            D 38440            E \*\*\*

35. 有多少种不同的方法可将英文字 INFORMATION 的字母重排列使得前面五个字母都是母音 (即 A、E、I、O、U) ?

In how many distinct ways can the letters in the word INFORMATION be rearranged such that the first five letters are all vowels (i.e., A, E, I, O, U)?

- A 43200            B 21600            C 10800            D 5400            E \*\*\*