雪兰莪暨吉隆坡福建会馆
新 纪 元 大 学 学 院
联合主办

ANJURAN BERSAMA
PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR
&
KOLEJ UNIVERSITI NEW ERA

第三十四届（2019 年度）
雪隆中学华罗庚杯数学比赛
PERTANDINGAN MATEMATIK PIALA HUA LO GENG ANTARA SEKOLAH-SEKOLAH MENENGAH DI NEGERI SELANGOR DAN KUALA LUMPUR YANG KE-34 (2019)

~~高中组~~
BAHAGIAN MENENGAH ATAS

日期 : 2019 年 7 月 7 日 （星期日）
Tarikh : 7 Julai 2019 (Hari Ahad)

时间 : 10:00→12:00（两小时）
Masa : 10:00→12:00（2 jam）

地点 : 新纪元大学学院黄迓莱活动中心
Tempat : Ng Ah Choo Multipurpose Hall, Kolej Universiti New Era
UG, Block C, Lot 5, Seksyen 10, Jalan Bukit,
43000 Kajang, Selangor

*****说明*****

1. 不准使用计算机。
2. 不必使用对数表。
3. 对一题得 4 分，错一题倒扣 1 分。
4. 答案 E：若是“以上皆非”或“不能确定”，一律以“***”代替之。

*****INSTRUCTIONS*****

1. Calculators are not allowed.
2. Logarithm table is not to be used.
3. 4 marks will be awarded for each correct answer and 1 mark will be deducted for each wrong answer.
4. (E) *** indicates “none of the above”.

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1. 下午三点四十五分时，时钟上的时针与分针所成的较小的角是多少度？
   At 3:45 in the afternoon, what is the smaller angle formed by the hour hand and the minute hand?
   A. $157.5°$ B. $165°$ C. $172.5°$ D. $180°$ E. ***

2. 已知 $\log_2 3 = a$，$\log_3 7 = b$，求 $\log_{21} 42$。
   Given that $\log_2 3 = a$, $\log_3 7 = b$, find $\log_{21} 42$.
   A. $1+a+ab$ B. $1+\dfrac{b}{1+a}$ C. $\dfrac{1+a}{b}$ D. $1+\dfrac{1}{a(1+b)}$ E. ***

3. 一个长方形的长减少了 20%，宽增加了 $x\%$，面积却保持不变。求 x 的值。
   The length of a rectangle is decreased by 20%, and the width is increased by $x\%$, but the area remains the same. Find the value of x.
   A. 20 B. 22.5 C. 25 D. 30 E. ***

4. 若 M 与 m 分别是满足不等式 $6n^2-5n\le 99$ 的最大与最小的整数，求 $M-m$ 的值。
   If M and m are respectively the largest and the smallest integers that satisfy the inequality $6n^2-5n\le 99$, find the value of $M-m$.
   A. 7 B. 8 C. 9 D. 10 E. ***

5. 一项数学比赛参赛的男生与女生的人数之比是 $3:2$。已知参赛者的 15% 得奖，而得奖的男生与得奖的女生的人数之比是 $2:1$，求没有得奖的男生与没有得奖的女生的人数之比。
   The ratio of male participants to female participants in a mathematics competition is $3:2$. Given that 15% of the participants are prize winners, and among the prize winners, the ratio of male to female is $2:1$. Find the ratio of male to female among the students that are not prize winners.
   A. $1:1$ B. $4:3$ C. $5:3$ D. $10:7$ E. ***

6. 已知函数 $f:\mathbb{R}\to\mathbb{R}$ 满足 $f(2x-1)=4x^2-8x+17$，求 $f(2x+1)$。

Given that the function $f:\mathbb{R}\to\mathbb{R}$ satisfies $f(2x-1)=4x^2-8x+17$. Find $f(2x+1)$.

A. $4x^2+8x+13$ B. $4x^2+17$ C. $4x^2-8x+19$ D. $4x^2+13$ E. ***

7. 已知曲线 $y=2x^2-19x+18$ 与直线 $y=x+k$ 不相交，求 k 的最大整数值。

Given that the curve $y=2x^2-19x+18$ does not intersect the line $y=x+k$, find the largest integer value of k.

A. -31 B. -32 C. -33 D. 31 E. ***

8. 求 2019 的正因子的个数。

Find the number of positive factors of 2019.

A. 2 B. 4 C. 8 D. 16 E. ***

9. 已知

$$N=625\left(1-\frac{9}{5^2}\right)\left(1-\frac{9}{8^2}\right)\left(1-\frac{9}{11^2}\right)\cdots\left(1-\frac{9}{125^2}\right)$$

求 N 的各位数字之和。

Given that

$$N=625\left(1-\frac{9}{5^2}\right)\left(1-\frac{9}{8^2}\right)\left(1-\frac{9}{11^2}\right)\cdots\left(1-\frac{9}{125^2}\right)$$

Find the sum of the digits of N.

A. 11 B. 12 C. 13 D. 14 E. ***

10. 甲班有48位学生，20位是女生。乙班有36位学生，24位是女生。从甲、乙两班分别任意选出两位学生。求四位选出的学生中至少有一位是女生的概率。

Among the 48 students in Class K, 20 of them are girls. Among the 36 students in Class Y, 24 of them are girls. Four students are randomly selected from these 2 classes, 2 from each class. Find the probability that among the 4 selected students, at least one of them is a girl.

A. $\frac{5}{18}$ B. $\frac{29}{36}$ C. $\frac{907}{940}$ D. $\frac{5485}{5922}$ E. ***

11. 求 $(1!\times1)+(2!\times2)+(3!\times3)+\ldots.+(100!\times100)$ 除以101的余数。

Find the remainder when $(1!\times1)+(2!\times2)+(3!\times3)+\ldots.+(100!\times100)$ is divided by 101.

A. 0 B. 1 C. 100 D. 101 E. ***

12. 小于1200的正整数中有多少个与60互质？

Among the positive integers less than 1200, how many of them are relatively prime to 60?

A. 160 B. 300 C. 320 D. 360 E. ***

13. 图1中，ΔABC 是等边三角形，$AB=6$。$\overset{\frown}{AB}$，$\overset{\frown}{BC}$，$\overset{\frown}{CA}$ 分别为以 C，A，B 为圆心的弧。求此图形的面积。

In the Figure 1, ΔABC is a regular triangle, $AB=6$. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CA}$ are circular arcs with centres at C, A and B respectively. Find the area of the figure.

A. $18\pi-18\sqrt{3}$ B. $18\pi-27\sqrt{3}$ C. $36\pi-27\sqrt{3}$
D. $27\pi-27\sqrt{3}$ E. ***

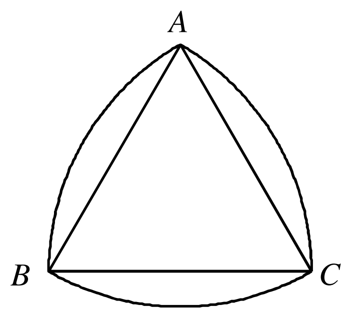


图 1
Figure 1

14. 已知数列 $\{a_n\}$ 的定义为 $a_1=2$，且对于所有 $n\geq 1$，$a_{n+1}=a_n+2n-1$。求 a_{100} 的最后两位数。

Given that the sequence $\{a_n\}$ is defined as $a_1=2$, and $a_{n+1}=a_n+2n-1$ for all $n\geq 1$. Find the last two digits of a_{100}.

A. 01 B. 02 C. 03 D. 92 E. ***

15. 已知 $N=1\times 2\times 3\times\cdots\times 500$ 是由1到500这500个正整数的乘积。若 N 可以被 6^k 整除，求 k 的最大可能值。

Given that $N=1\times 2\times 3\times\cdots\times 500$ is the product of the positive integers from 1 to 500. If N is divisible by 6^k, find the largest possible value of k.

A. 98 B. 247 C. 248 D. 494 E. ***

16. 图2中，$ABCD$ 是长方形，其面积为1296。已知 $DM=MC$，$CN=2BN$，对角线 BD 与直线 AN 及 AM 分别相交于 P 及 Q 两点，直线 EF 经过点 Q 而与直线 AD 平行，并与直线 AN 相交于点 R。求 ΔPQR 的面积。

In the Figure 2, $ABCD$ is a rectangle with area 1296. Given that $DM=MC$, $CN=2BN$. The diagonal BD intersects the line AN and AM at points P and Q respectively. The line EF passes through the point Q and is parallel to the line AD. It intersects the line AN at the point R. Find the area of ΔPQR.

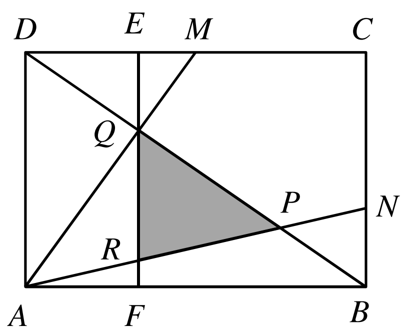


图 2
Figure 2

A. 150 B. 160 C. 175 D. 180 E. ***

17. 甲，乙，丙，丁，戊5人参加一项游戏，他们得奖的概率分别为$\frac{6}{30}$，$\frac{5}{30}$，$\frac{4}{30}$，$\frac{3}{30}$，$\frac{2}{30}$。他们每个人都没有获奖的概率至少是多少？

Five persons *A*, *B*, *C*, *D*, *E* participate in a game. The chances for each of them to win a prize are $\frac{6}{30}$, $\frac{5}{30}$, $\frac{4}{30}$, $\frac{3}{30}$ and $\frac{2}{30}$ respectively. The probability that none of them win a prize is at least how much?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. ***

18. 一直圆柱的底圆半径是1，高是5。若一球的表面积是此圆柱体的表面积的三倍，求圆柱体的体积与球的体积之比。

The radius of the base of a right circular cylinder is 1, and the height of the cylinder is 5. If the surface area of a ball is three times the surface area of this cylinder, find the ratio of the volume of the cylinder to the volume of the ball.

A. $\frac{5}{12}$ B. $\frac{5}{24}$ C. $\frac{5}{36}$ D. $\frac{15}{32}$ E. ***

19. 图3中，求$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7$。

In the Figure 3, find $\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7$.

A. 450° B. 540° C. 630°
D. 720° E. ***

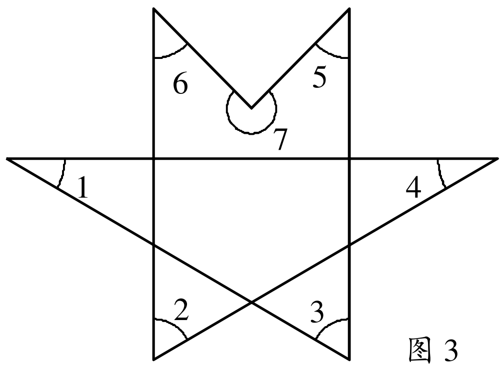


图 3
Figure 3

20. 图4中，$A(0,4)$，$B(3,0)$，$C(8,0)$，$D(h,k)$四点围出一个等腰梯形，$AB//CD$。求$5(k-h)$的值。

In the Figure 4, *ABCD* is an isosceles trapezoid enclosed by the four points $A(0,4)$, $B(3,0)$, $C(8,0)$, $D(h,k)$, and $AB//CD$. Find the value of $5(k-h)$.

A. 34 B. 35 C. 36
D. 37 E. ***

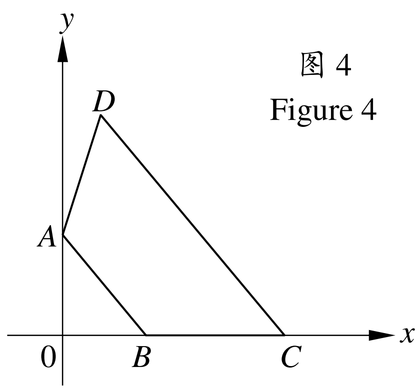


图 4
Figure 4

21. 已知 ω 是一复数使得 $\omega^7=1$ 但 $\omega\neq 1$。求 $(2+\omega)(2+\omega^2)(2+\omega^3)(2+\omega^4)(2+\omega^5)(2+\omega^6)$ 的值。

Given that ω is a complex number such that $\omega^7=1$ but $\omega\neq 1$. Find the value of $(2+\omega)(2+\omega^2)(2+\omega^3)(2+\omega^4)(2+\omega^5)(2+\omega^6)$

A. 43 B. 127 C. 129 D. -129 E. ***

22. 求小于 $(7+4\sqrt{3})^3$ 的最大整数。

Find the largest integer smaller than $(7+4\sqrt{3})^3$

A. 2700 B. 2701 C. 2702 D. 2703 E. ***

23. 图5中，$ABCD$ 是长方形，$AB=5$，$BC=4$。直线 AP 与直线 QC 平行，它们之间的距离为 h。若平行四边形 $APCQ$ 的面积与长方形 $ABCD$ 的面积之比是 $3:8$，求 $120h^2$ 的值。

In the Figure 5, $ABCD$ is a rectangle, $AB=5$, $BC=4$. The lines AP and QC are parallel, with distance h apart. If the ratio of the area of parallelogram $APCQ$ to the area of the rectangle $ABCD$ is $3:8$, find the value of $120h^2$.

A. 162 B. 200 C. 216

D. 224 E. ***

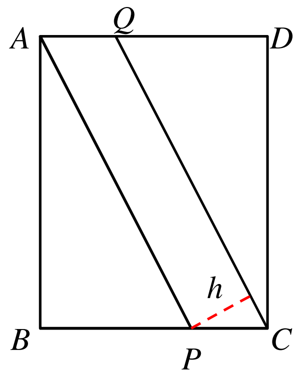


图 5
Figure 5

24. 有多少个数字可重复的正的5位数 $\overline{x_1x_2x_3x_4x_5}$ 满足 $x_1\leq x_2\leq x_3\leq x_4\leq x_5$？

How many 5-digit positive integers $\overline{x_1x_2x_3x_4x_5}$ are there such that $x_1\leq x_2\leq x_3\leq x_4\leq x_5$?

A. 715 B. 1001 C. 1287 D. 2002 E. ***

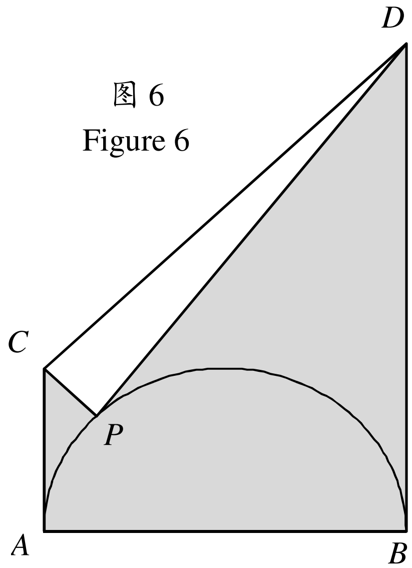
25. 如图6所示，AB 为半圆的直径，AC 与 BD 垂直于 AB，P 为半圆上的一点。已知 $AB=6$，$AC=3$，$BD=9$，求五边形 $ABDPC$ 的面积的最大可能值。

As shown in the Figure 6, AB is the diameter of the semicircle. AC and BD are perpendicular to AB. P is a point on the semicircle. Given that $AB=6$, $AC=3$, $BD=9$, find the largest possible value of the area of pentagon $ABDPC$.

A. 30 B. 27 C. $18+9\sqrt{2}$

D. $18\sqrt{2}$ E. ***

图 6
Figure 6



26. 已知x，y是实数且$x^2-y^2=32$，$(x+y)^4+(x-y)^4=4352$，求x^2+y^2的值。

Given that x, y are real numbers such that $x^2-y^2=32$ and $(x+y)^4+(x-y)^4=4352$, find the value of x^2+y^2.

A. 40　B. 48　C. 80　D. 96　E. ***

27. 若(x, y)满足方程组$\begin{cases}|x|-x-y+2=0\\|y|+y+5x=1\end{cases}$，求$x+y$的值。

If (x, y) satisfies the system of equations $\begin{cases}|x|-x-y+2=0\\|y|+y+5x=1\end{cases}$, find the value of $x+y$.

A. 3　B. 5　C. 7　D. 9　E. ***

28. 图7中，$A_1B_1C_1$是等边三角形。A_2，B_2，C_2分别是A_1B_1，B_1C_1，C_1A_1边上的点使得$A_1A_2=B_1B_2=C_1C_2=\frac{1}{5}A_1B_1$，$A_3$，$B_3$，$C_3$分别是$A_2B_2$，$B_2C_2$，$C_2A_2$边上的点使得$A_2A_3=B_2B_3=C_2C_3=\frac{1}{5}A_2B_2$。重复这个过程以构造出无限个三角形$\Delta A_1B_1C_1$，$\Delta A_2B_2C_2$，$\Delta A_3B_3C_3$，…。若$S_n$为$\Delta A_nB_nC_n$的面积，求$\frac{\sum_{n=1}^{\infty}S_n}{S_1}=\frac{S_1+S_2+S_3+\cdots}{S_1}$的值。

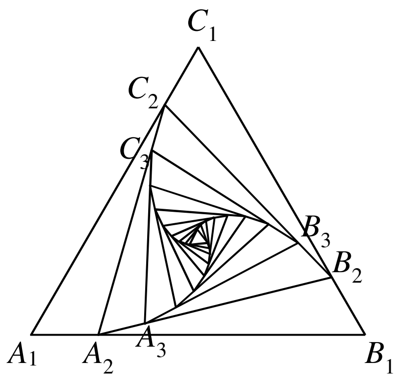


图 7
Figure 7

In the Figure 7, $A_1B_1C_1$ is a regular triangle. A_2, B_2, C_2 are points on A_1B_1, B_1C_1, C_1A_1 respectively such that $A_1A_2=B_1B_2=C_1C_2=\frac{1}{5}A_1B_1$. A_3, B_3, C_3 are points on A_2B_2, B_2C_2, C_2A_2 respectively such that $A_2A_3=B_2B_3=C_2C_3=\frac{1}{5}A_2B_2$. Repeat this process to construct infinitely many triangles $\Delta A_1B_1C_1$, $\Delta A_2B_2C_2$, $\Delta A_3B_3C_3$, …. If S_n is the area of $\Delta A_nB_nC_n$, find the value of $\frac{\sum_{n=1}^{\infty}S_n}{S_1}=\frac{S_1+S_2+S_3+\cdots}{S_1}$.

A. $\frac{38}{25}$　B. 2　C. $\frac{25}{12}$　D. 3　E. ***

29. 有多少个由数字1至9所组成的六位正整数，其每个数字出现至少两次？（例如：121233，122221, 222222等都是这样的六位数。）

How many 6-digit positive integers which are formed by the digits 1 to 9, are such that each of the digits in the number appears at least twice? (For example, 121233, 122221, 222222 are such 6-digit numbers.)

A. 9369　B. 8829　C. 8649　D. 7569　E. ***

30. 已知x、y、z是实数使得$x+y+z=0$及$xyz=-432$。若$a=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$，求$a$的最小可能值。

Given that x, y, z are real numbers such that $x+y+z=0$ and $xyz=-432$. If $a=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$, find the smallest possible value of a.

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. 1 E. ***

31. 有多少个正整数x满足$\log_{\frac{x}{8}}\frac{x^2}{4}<7+\log_2\frac{8}{x}$?

How many positive integers x satisfy $\log_{\frac{x}{8}}\frac{x^2}{4}<7+\log_2\frac{8}{x}$?

A. 118 B. 119 C. 120 D. 121 E. ***

32. 图8中，O是圆心，AB是直径。C是半径OA上的一点，D是圆上的一点使得CD垂直于AB，E是线段BD上的一点使得CE垂直于BD。已知圆的半径为34，$OE=30$，求CE的长。

In the Figure 8, O is the center of the circle, AB is a diameter. C is a point on OA. D is a point on the circle such that CD is perpendicular to AB. E is a point on the line segment BD such that CE is perpendicular to BD. Given that the radius of the circle is 34, and $OE=30$, find the length of CE.

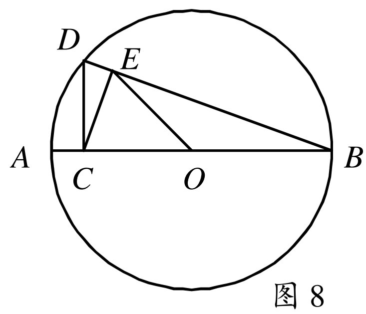


图 8
Figure 8

A. 30 B. 24 C. 19 D. 16 E. ***

33. 有多少个正整数n使得$\sqrt{n^2+124n}$也是整数?

How many positive integers n are there such that $\sqrt{n^2+124n}$ is also an integer?

A. 0 B. 1 C. 2 D. 4 E. ***

34. 图9所示，P是正方形$ABCD$内的一点。已知$AP=7$，$BP=6$，$CP=11$，$\angle APB=\theta$，求$\tan\theta$。

As shown in the Figure 9, P is a point in the square $ABCD$. Given that $AP=7$, $BP=6$, $CP=11$, $\angle APB=\theta$, find $\tan\theta$.

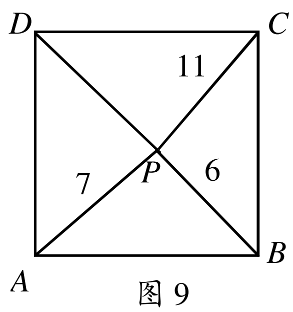


图 9
Figure 9

A. $-\frac{1}{3}$ B. $-\frac{1}{2}$ C. $-\frac{2}{3}$

D. -1 E. ***

35. 求$1\times3\times5\times7\times......\times2017\times2019$除以1000的余数。

Find the remainder when $1\times3\times5\times7\times......\times2017\times2019$ is divided by 1000.

A. 125 B. 375 C. 625 D. 875 E. ***

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