雪兰莪暨吉隆坡福建会馆
新 纪 元 大 学 学 院

联合主办

ANJURAN BERSAMA
PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR
&
KOLEJ UNIVERSITI NEW ERA

第三十六届（2024年度）
雪隆森中学华罗庚杯数学比赛
PERTANDINGAN MATEMATIK PIALA HUA LO GENG
ANTARA SEKOLAH-SEKOLAH MENENGAH
DI NEGERI SELANGOR, KUALA LUMPUR DAN NEGERI SEMBILAN
YANG KE-36 (2024)

～～初中组～～
BAHAGIAN MENENGAH BAWAH

日期 : 2024 年8 月18 日（星期日）
Tarikh : 18 Ogos 2024 (Hari Ahad)

时间 : 10:00 – 12:00（两小时）
Masa : 10:00 – 12:00 (2 jam)

地点 : 新纪元大学学院黄迂茱活动中心
Tempat : Ng Ah Choo Multipurpose Hall, Kolej Universiti New Era
UG, Block C, Lot 5, Seksyen 10, Jalan Bukit,
43000 Kajang, Selangor

*** 说明***

1. 不准使用计算机。
2. 对一题得4 分，错一题倒扣1 分。
3. E. *** 表示“以上皆非”。

*****INSTRUCTIONS*****

1. Calculators are not allowed.
2. 4 marks will be awarded for each correct answer but 1 mark will be deducted for each wrong answer.
3. E. *** indicates “none of the above”.

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1. 求最接近

$$\frac{3^{2020}+3^{2024}}{3^{2020}+3^{2022}}$$

的整数值。

Find the integer value that is nearest to

$$\frac{3^{2020}+3^{2024}}{3^{2020}+3^{2022}}.$$

A. 8　　B. 9　　C. 10　　D. 11　　E. ***

2. 以下哪个数的值最大?

Which of the following numbers is the largest?

A. $\sqrt{2}$　　B. $\sqrt[3]{3}$　　C. $\sqrt[4]{4}$　　D. $\sqrt[5]{5}$　　E. ***

3. 若一钝角三角形的两边长分别为 21 和 23, 以下哪个可能是第三边的长?

Suppose that two sides of an obtuse triangle are 21 and 23. Which of the followings can be the length of the third side?

A. 10　　B. 20　　C. 30　　D. 40　　E. ***

4. 设 $x \diamond y = \dfrac{x-y}{1+x^2+2y^2}$。求 $\dfrac{1}{2} \diamond \left[\left(\dfrac{1}{3} \diamond \dfrac{1}{4}\right) \diamond \left(\dfrac{1}{3} \diamond \dfrac{1}{4}\right)\right]$ 的值。

Let $x \diamond y = \dfrac{x-y}{1+x^2+2y^2}$. Find the value of $\dfrac{1}{2} \diamond \left[\left(\dfrac{1}{3} \diamond \dfrac{1}{4}\right) \diamond \left(\dfrac{1}{3} \diamond \dfrac{1}{4}\right)\right]$.

A. $\dfrac{1}{5}$ B. $\dfrac{2}{5}$ C. $\dfrac{3}{5}$ D. $\dfrac{4}{5}$ E. ***

5. 已知 x 与 y 是整数，且满足 $xy + 4x - 2y = 0$。求所有此 x 之和。

Given that x and y are integers satisfying $xy + 4x - 2y = 0$. Find the sum of all such x.

A. 13 B. 14 C. 15 D. 16 E. ***

6. 设 N 为一个由从 1 开始的整数序列依次连接形成的 2024 位整数（即 $N = 123456789101112\ldots$）。求 N 被 5 除的余数。

Let N be a 2024−digit integer formed by concatenating the sequence of whole numbers beginning from 1 (i.e., $N = 123456789101112\ldots$). Find the remainder when N is divided by 5.

A. 1 B. 2 C. 3 D. 4 E. ***

7. 对于正整数 n, n 的阶乘定义为 $n! = n(n-1)(n-2)\cdots(2)(1)$。若 $m! = \dfrac{(4!)!}{4!}$，求整数 m 之值。

For positive integer n, the factorial of n is $n! = n(n-1)(n-2)\cdots(2)(1)$. Suppose $m! = \dfrac{(4!)!}{4!}$, find the integer value m.

A. 3 B. 6 C. 12 D. 23 E. ***

8. 求所有使得 $(x^2 - 7x + 11)^{x+1} = 1$ 成立的整数 x 之和。

Find the sum of all integers x such that $(x^2 - 7x + 11)^{x+1} = 1$.

A. 9 B. 8 C. 7 D. 6 E. ***

9. 求小于$\left(5+\sqrt{21}\right)^2$的最大整数值。
Determine the largest integer that is less than $\left(5+\sqrt{21}\right)^2$.

A. 90　　B. 91　　C. 92　　D. 93　　E. ***

10. 已知 x 与 y 是实数，求 $x^2y^2+4y^2+4xy-4y+6$ 的最小值。
Given that x and y are real numbers, find the minimum value of $x^2y^2+4y^2+4xy-4y+6$.

A. 1　　B. 2　　C. 3　　D. 6　　E. ***

11. $84=2^2\times3\times7$ 有 3 个质因数，2024 有几个质因数?
$84=2^2\times3\times7$ has 3 prime divisors, how many prime divisors does 2024 have?

A. 3　　B. 4　　C. 5　　D. 6　　E. ***

12. 设 m 与 n 为正整数，且满足

$$\frac{m}{n}=0.ABCABCABC\ldots=0.\overline{ABC}$$

其中 A,B 及 C 为相异的非零数字。求 $m+n$ 的最小值。
Suppose m and n are positive integers such that

$$\frac{m}{n}=0.ABCABCABC\ldots=0.\overline{ABC}$$

where A,B and C are distinct non-zero digits. Find the minimum value of $m+n$.

A. 31　　B. 32　　C. 41　　D. 42　　E. ***

13. 有多少个整数 n 使得 $f(n)=n^2+4n-36$ 是一个完全平方数?
How many integer values n such that $f(n)=n^2+4n-36$ is a perfect square?

A. 1　　B. 2　　C. 3　　D. 4　　E. ***

14. 已知 $f(x) = 4x - 4$, 且$f\left(a^2\right) = (f(a))^2$。求 a 的最大可能值。

Given that $f(x) = 4x - 4$ and $f\left(a^2\right) = (f(a))^2$. Find the largest possible value of a.

A. 1 B. $\frac{5}{3}$ C. 2 D. $\frac{5}{2}$ E. ***

15. 若两个数的最小公倍数（LCM）为 84, 最大公因数（GCD）为 12, 以下哪个是这两个数之和?

If two numbers have the least common multiple (LCM) of 84 and greatest common divisor (GCD) of 12, which of the followings is the sum of these two numbers?

A. 60 B. 72 C. 96 D. 108 E. ***

16. 已知 $\sqrt{17 \times 20 \times 23 \times 26 + 81}$ 为一整数，求此整数的各位数字之和。例： 123 的各位数字之和为 $1 + 2 + 3 = 6$。

Given that $\sqrt{17 \times 20 \times 23 \times 26 + 81}$ is an integer, find the sum of the digits of this integer. For instance, the sum of the digits of 123 is $1 + 2 + 3 = 6$.

A. 7 B. 8 C. 9 D. 10 E. ***

17. 若 $4x^2 + x + 2 = 0$, 求 $8x^3 - 10x^2 + x + 18$ 之值。

If $4x^2 + x + 2 = 0$, find the value of $8x^3 - 10x^2 + x + 18$.

A. 21 B. 22 C. 23 D. 24 E. ***

18. 求方程

$$1 + \log\sqrt{1+x} + 3\log\sqrt{1-x} = \log\sqrt{1-x^2}$$

的解。

Find the solution to the equation

$$1 + \log\sqrt{1+x} + 3\log\sqrt{1-x} = \log\sqrt{1-x^2}.$$

A. $\frac{1}{10}$ B. $\frac{9}{10}$ C. $\frac{1}{11}$ D. $\frac{10}{11}$ E. ***

19. 六位数 $20AB24$ 及 $6A30BC$ 皆是 6的倍数。以下哪个可能是 C 的值?

The six-digit numbers $20AB24$ and $6A30BC$ are both multiples of 6. Which of the followings could be a value of C?

A. 0　　B. 2　　C. 4　　D. 6　　E. ***

20. 求所有至少有一个数字为 1 的两位数的总和。

Find the sum of all two-digit numbers that at least one of the digits is 1.

A. 571　　B. 582　　C. 593　　D. 604　　E. ***

21. 设 $x,\ y$ 为正整数使得 $\dfrac{4}{23} < \dfrac{x}{y} < \dfrac{2}{11}$。求 x 的最小值。

Suppose $x,\ y$ are positive integers such that $\dfrac{4}{23} < \dfrac{x}{y} < \dfrac{2}{11}$. Find the minimum value of x.

A. 3　　B. 6　　C. 21　　D. 45　　E. ***

22. 求最接近 $\sqrt{52+14\sqrt{3}}+\sqrt{52-14\sqrt{3}}$ 的整数值。

Find the integer value that is nearest to $\sqrt{52+14\sqrt{3}}+\sqrt{52-14\sqrt{3}}$.

A. 12　　B. 13　　C. 14　　D. 15　　E. ***

23. 若 $a,\ b,\ c,\ d$ 为相异的正整数，且满足 $a^b = c^d$, 求 $a+b+c+d$ 的最小值。

Suppose $a,\ b,\ c,\ d$ are distinct positive integers such that $a^b = c^d$, find the minimum value of $a+b+c+d$.

A. 13　　B. 14　　C. 15　　D. 16　　E. ***

24. 设 $a,\ b,\ c$ 为实数。已知函数 $y = ax^2+bx+c$ 的图像经过点 $\left(0, 1+\sqrt{3}\right)$, $\left(-1, 2\sqrt{2}\right)$ 及 $\left(1-\sqrt[3]{3}, 0\right)$。求 $a-b+c$ 的值。

Let $a,\ b,\ c$ be real numbers. Given that the graph of $y = ax^2+bx+c$ passes through the points $\left(0, 1+\sqrt{3}\right)$, $\left(-1, 2\sqrt{2}\right)$ and $\left(1-\sqrt[3]{3}, 0\right)$. Find the value of $a-b+c$.

A. $\sqrt{2}$　　B. $\sqrt[3]{3}$　　C. $\sqrt{3}$　　D. $2\sqrt{2}$　　E. ***

25. 一直角三角形的周长为 34, 其边长的乘积为 544。求此三角形的面积。

A right-angled triangle has perimeter 34 and the product of its side lengths is 544. Find the area of the triangle.

A. 15 B. 16 C. 17 D. 18 E. ***

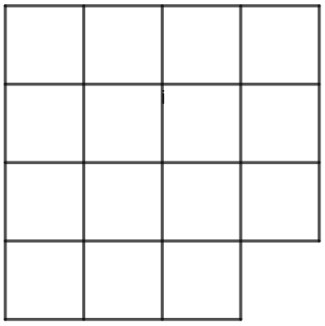
26. 一四边形 $ABCD$ 的边长为 $AB = 3$, $BC = 4$, $CD = 13$ 及 $DA = 12$。已知对角线 $AC = 5$, 求四边形 $ABCD$ 面积的最小可能值。

In the quadrilateral $ABCD$, the side lengths are $AB = 3$, $BC = 4$, $CD = 13$, and $DA = 12$. Given that the diagonal $AC = 5$, find the smallest possible area of quadrilateral $ABCD$.

A. 20 B. 24 C. 30 D. 36 E. ***

27. 右图中有多少个长方形?

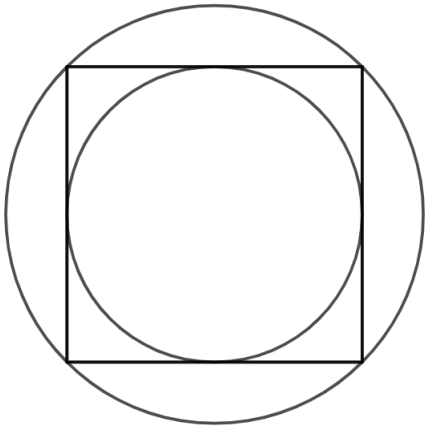
How many rectangles are there in the figure on the right?



A. 81 B. 84 C. 86
D. 90 E. ***

28. 如右图所示，一正方形内切于一圆，另一圆内切于此正方形。求外圆的面积与内圆的面积之比。

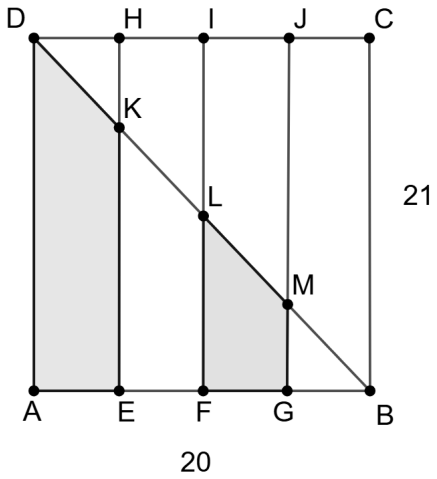
As shown in the figure on the right, a square is inscribed in a circle, and the other circle is inscribed within the square. Find the ratio of the area of the outer circle to the area of the inner circle.



A. $\sqrt{2}$ B. $\sqrt{3}$ C. 2
D. $\sqrt{5}$ E. ***

29. 右图中，$ABCD$ 为一个边长为 $AB = 20$ 及 $BC = 21$ 的长方形。点 $E,\ F,\ G$ 在边 AB 上，使得 $AE = EF = FG = GB$。点 $H,\ I,\ J$ 在边 DC 上，使得 $DH = HI = IJ = JC$。对角线 DB 与 $EH,\ FI,\ GJ$ 分别相交于点 $K,\ L,\ M$。求两个阴影梯形 $AEKD$ 及 $FGML$ 的总周长。

In the figure on the right, $ABCD$ is a rectangle with side lengths $AB = 20$ and $BC = 21$. Points $E,\ F,\ G$ are on AB such that $AE = EF = FG = GB$. Similarly, points $H,\ I,\ J$ are on DC such that $DH = HI = IJ = JC$. The diagonal DB intersects $EH,\ FI,\ GJ$ at points $K,\ L,\ M$ respectively. Find the total perimeter of the two shaded trapeziums $AEKD$ and $FGML$.

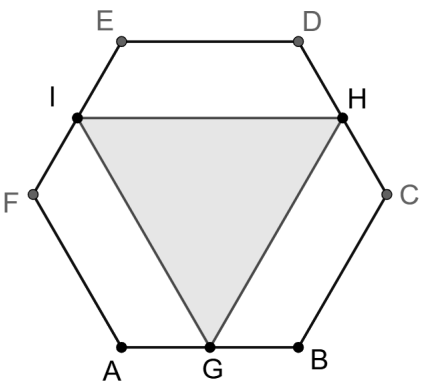


A. 75　　B. 76　　C. 77

D. 78　　E. ***

30. 右图中，$ABCDEF$ 为一正六边形。点 $G,\ H,\ I$ 分别是边 $AB,\ CD,\ EF$ 的中点。已知 $\triangle GHI$ 的面积为 90，求六边形 $ABCDEF$ 的面积。

In the figure on the right, $ABCDEF$ is a regular hexagon. Points $G,\ H,\ I$ are the midpoints of $AB,\ CD,\ EF$ respectively. Given that the area of $\triangle GHI$ is 90, find the area of the hexagon $ABCDEF$.

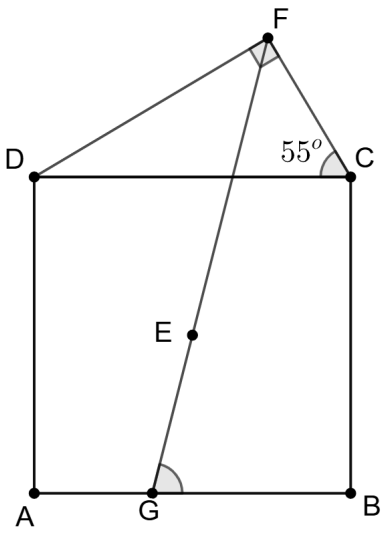


A. 210　　B. 220　　C. 230

D. 240　　E. ***

31. 右图中，$ABCD$ 为一正方形，点 E 是此正方形的中心。另外，CDF 为一直角三角形，其中 $\angle CFD = 90°$, $\angle FCD = 55°$。点 G 位于边 AB 上使得 GEF 成一直线。求 $\angle FGB$。

In the figure on the right, $ABCD$ is a square with point E as its centre. In addition to that, CDF is a right-angled triangle where $\angle CFD = 90°$ and $\angle FCD = 55°$. Point G is on AB such that GEF forms a straight line. Find $\angle FGB$.

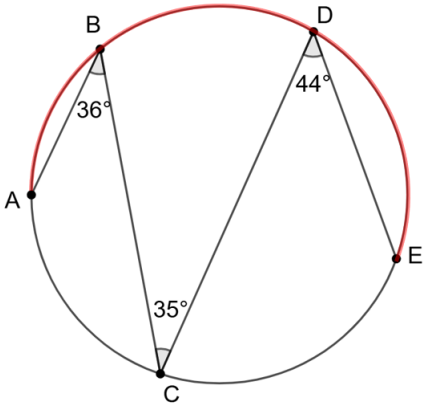


A. $80°$　　B. $78°$　　C. $76°$

D. $75°$　　E. ***

32. 如右图所示，点 A, B, C, D, E 位于圆周上，其中 $\angle ABC = 36°$, $\angle BCD = 35°$, $\angle CDE = 44°$。若圆的半径为 18, 求优弧 $\overset{\frown}{AE}$ 的长。

As shown in the figure on the right, points A, B, C, D, E lie on the circumference of the circle, where $\angle ABC = 36°$, $\angle BCD = 35°$ and $\angle CDE = 44°$. If the circle is of radius 18, find the length of the major arc $\overset{\frown}{AE}$.

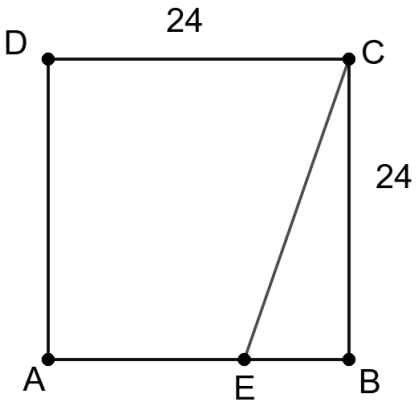


A. 22π　　B. 21π　　C. 20π

D. 19π　　E. ***

33. 右图中，$ABCD$ 是一边长为 24 的正方形。点 E 位于边 AB 上，使得 $\triangle BCE$ 的面积与四边形 $AECD$ 的面积之比为 $1:5$。求 AE 的长。

In the figure on the right, $ABCD$ is a square of side length 24. Point E is on AB such that the ratio of the area of $\triangle BCE$ to the area of quadrilateral $AECD$ is $1:5$. Find the length of AE.

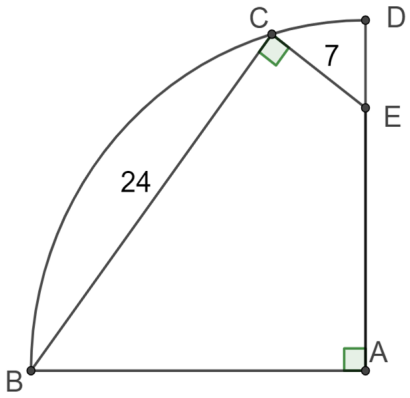


A. 12　　B. 14　　C. 16

D. 20　　E. ***

34. 右图中，ABD 为一四分之一圆。其中，点 C 位于弧 $\overset{\frown}{BD}$ 上，点 E 位于线段 AD 上，使得 $BC = 24$, $CE = 7$。求 AE 的长。

In the figure on the right, ABD is a quarter circle. Point C is on arc $\overset{\frown}{BD}$ while point E is on line segment AD such that $BC = 24$ and $CE = 7$. Find the length of AE.

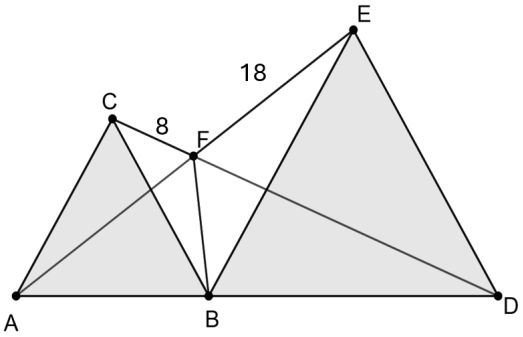


A. 12　　B. 13　　C. 14

D. 15　　E. ***

35. 右图中，ABC 与 BDE 为等边三角形，且 ABD 为一直线。线段 CD 与 AE 相交于点 F。已知 $CF = 8$, $FE = 18$，求 BF 的长。

In the figure on the right, ABC and BDE are equilateral triangles such that ABD forms a straight line. The line segments CD and AE intersect at point F. Given that $CF = 8$ and $FE = 18$, find the length of BF.



A. $6\sqrt{3}$　　B. 11　　C. 12

D. $7\sqrt{3}$　　E. ***

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