

雪兰莪暨吉隆坡福建会馆
新纪元大学学院

联合主办

ANJURAN BERSAMA
PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR
&
KOLEJ UNIVERSITI NEW ERA

第三十七届（2025年度）
雪隆森中学华罗庚杯数学比赛
PERTANDINGAN MATEMATIK PIALA HUA LO GENG
ANTARA SEKOLAH-SEKOLAH MENENGAH
DI NEGERI SELANGOR, KUALA LUMPUR DAN NEGERI SEMBILAN
YANG KE-37 (2025)

~~高中组~~
BAHAGIAN MENENGAH ATAS

日期 : 2025年6月22日 (星期日)
Tarikh : 22 Jun 2025 (Hari Ahad)

时间 : 10:00 – 12:00 (两小时)
Masa : 10:00 – 12:00 (2 jam)

地点 : 吉隆坡中华独立中学 – 光前堂
Tempat : Sekolah Menengah Persendirian Chong Hwa Kuala Lumpur
Jalan St. Thomas, Batu 3 1/2 off Jalan Sultan Azlan Shah,
51100 Kuala Lumpur.

*** 说明***

1. 不准使用计算机。
2. 对一题得 4 分，错一题倒扣 1 分。
3. E. *** 表示“以上皆非”。

INSTRUCTIONS

1. Calculators are not allowed.
2. 4 marks will be awarded for each correct answer but 1 mark will be deducted for each wrong answer.
3. E. *** indicates “none of the above”.

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1. 一长方体盒子的长、宽、高各为 18, 6 及 12。求能完全装进此盒子里的最大球的体积。

A rectangular box has length 18, width 6 and height 12. Find the volume of the largest ball that can fit completely inside this box.

- A. 36π B. 108π C. 144π D. 216π E. ***

2. 已知 $P(11, 8)$, $Q(a, -4)$ 两点到圆 $x^2 + y^2 + 4x - 14y = 0$ 的距离相等, 且 $a > 0$ 。求 a 的值。

Given that two points $P(11, 8)$ and $Q(a, -4)$ are equidistant to the circle $x^2 + y^2 + 4x - 14y = 0$, where $a > 0$. Find the value of a .

- A. 5 B. 7 C. 9 D. 11 E. ***

3. 已知方程式 $x^3 + x^2 - 5x - 1 = 0$ 的三个根为 α , β 及 γ , 方程式 $x^3 + px^2 + qx + r = 0$ 的三个根为 $\alpha + 2$, $\beta + 2$ 及 $\gamma + 2$ 。求 r 的值。

Given that the three roots of equation $x^3 + x^2 - 5x - 1 = 0$ are α , β and γ , and the three roots of equation $x^3 + px^2 + qx + r = 0$ are $\alpha + 2$, $\beta + 2$ and $\gamma + 2$. Find the value of r .

- A. -5 B. -1 C. 1 D. 5 E. ***

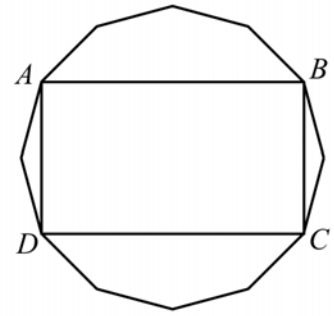
4. 设 n 是一大于 2025 的正整数, a_1, a_2, \dots, a_n 是一等差数列。若 $a_{1986} + a_{2025} = 100$ 且 $a_1 + a_n = 100$, 求 n 的值。

Let n be a positive integer larger than 2025, and let a_1, a_2, \dots, a_n be an arithmetic sequence. If $a_{1986} + a_{2025} = 100$ and $a_1 + a_n = 100$, find the value of n .

- A. 4000 B. 4001 C. 4010 D. 4011 E. ***

5. 右图所示为一正十二边形及其四个顶点 A, B, C, D 。若此正十二边形的每个边长为 1, 求长方形 $ABCD$ 的面积。

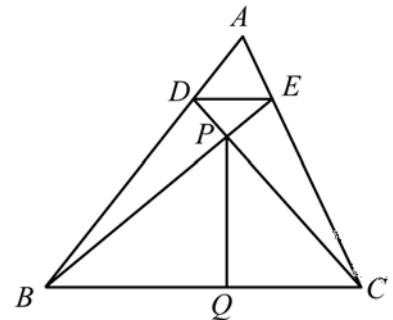
The figure on the right shows a regular 12-gon and its four vertices A, B, C, D . If the length of each side of this 12-gon is 1, find the area of rectangle $ABCD$.



- A. $6\sqrt{3} - 3$ B. $1 + 3\sqrt{3}$ C. $3 + 2\sqrt{3}$
D. $3 + 4\sqrt{3}$ E. ***

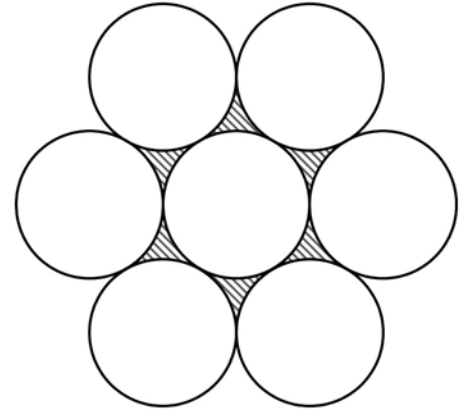
6. 右图所示的 $\triangle ABC$ 中, 点 D, E 分别在 AB, AC 边上, 且 $AD : AB = AE : AC = 1 : 6$ 。直线 CD 与 BE 相交于点 P 。点 Q 在 BC 边上使得 PQ 垂直于 BC 。已知 $\triangle ABC$ 的面积为 84 及 $BC = 12$, 求 PQ 的长。

In $\triangle ABC$ shown on the right, D, E are points lying on AB, AC respectively such that $AD : AB = AE : AC = 1 : 6$. Lines CD and BE intersect at point P , and Q is a point lying on BC such that PQ is perpendicular to BC . Given that the area of $\triangle ABC$ is 84 and $BC = 12$, find the length of PQ .



- A. 9 B. 10 C. 12
D. 14 E. ***

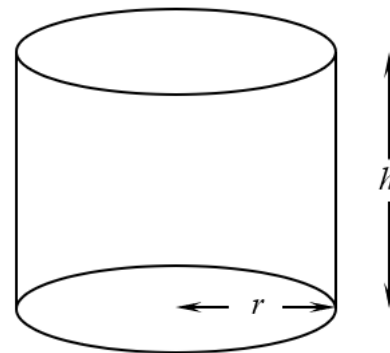
7. 右图中，中间圆的半径为 1，外围六个圆的半径相同。外围的圆分别与中间的圆外切，且与其两个相邻的圆也外切。求阴影部分的面积。



In the figure on the right, the circle in the center has radius 1 and the six circles in the outer rim have equal radii. Each circle in the outer rim is tangent to the circle in the center, and also tangent to its two neighboring circles. Find the area of the shaded region.

- A. $6\sqrt{3} - 3\pi$ B. $2\pi - 3\sqrt{3}$ C. $6\pi - 10\sqrt{3}$
 D. $\frac{\pi}{3}$ E. ***
8. 求 $(1 + x + x^2)^8$ 的展开式中 x^4 项的系数。
 Find the coefficient of x^4 in the expansion of $(1 + x + x^2)^8$.
- A. 70 B. 238 C. 258 D. 266 E. ***
9. 某中学的初中一有甲、乙、丙、丁四班，各有 53, 50, 49 及 48 人。对比期末考与期中考的数学成绩，甲、乙、丙、丁四班的平均分数分别提高了 1 分， x 分，5 分及 4 分。若整个初中一的平均分数提高了 3.2 分，求 x 的值。
 A high school has four form-one classes, A, B, C, D, which have 53, 50, 49, and 48 students in the class respectively. Comparing their final examination results to their midterm test results in mathematics, it is found that the averages of the four classes A, B, C, D have increased by 1, x , 5, and 4 points respectively. Suppose the overall average of all form-one students has increased by 3.2 points, find the value of x .
- A. 2.7 B. 2.8 C. 2.9 D. 3.0 E. ***

10. 如右图所示，一圆柱体的高为 h ，底圆半径为 r 。若 h 减少了 20%， r 减少了 10%，则圆柱体的体积减少了多少？



- A. 30% B. 35.2% C. 40%
D. 42.4% E. ***

11. 小于 1000 的正整数中，有多少个与 35 互质？

Among all positive integers less than 1000, how many of them are relatively prime to 35?

- A. 657 B. 658 C. 685 D. 686 E. ***

12. 有多少个整数 x 满足不等式 $x^4 + 9x^3 + 14x^2 \leq 0$?

How many integers x satisfy the inequality $x^4 + 9x^3 + 14x^2 \leq 0$?

- A. 7 B. 6 C. 5 D. 4 E. ***

13. 当 n 及 k 是正整数且 $k \leq n$ 时，定义 $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ 。求

$$\sum_{k=1}^{19} \binom{20}{k} \binom{20}{k+1} = \binom{20}{1} \binom{20}{2} + \binom{20}{2} \binom{20}{3} + \cdots + \binom{20}{19} \binom{20}{20}$$

的值。

For positive integers n and k where $k \leq n$, define $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Find the value of

$$\sum_{k=1}^{19} \binom{20}{k} \binom{20}{k+1} = \binom{20}{1} \binom{20}{2} + \binom{20}{2} \binom{20}{3} + \cdots + \binom{20}{19} \binom{20}{20}.$$

- A. $\binom{40}{19} - 1$ B. $\binom{40}{19} - 20$ C. $\binom{40}{20}$ D. $\binom{40}{20} - 1$ E. ***

14. 已知 $90 < x < 270$ ，且 $\tan x^\circ = \frac{\sin 142^\circ}{1 + \cos 38^\circ}$ 。求 x 的值。

Given that $90 < x < 270$ and $\tan x^\circ = \frac{\sin 142^\circ}{1 + \cos 38^\circ}$. Find the value of x .

- A. 142 B. 161 C. 199 D. 218 E. ***

15. 设函数 $f(x) = ax^2 + bx + c$ 。若曲线 $y = f(x)$ 经过三点 $P(-h, y_0)$ ， $Q(0, y_1)$ 及 $R(h, y_2)$ ，其中 $h \neq 0$ ，求积分 $\frac{1}{h} \int_{-h}^h f(x) dx$ 的值。

Consider the function $f(x) = ax^2 + bx + c$. Suppose the curve $y = f(x)$ passes through three points $P(-h, y_0)$, $Q(0, y_1)$ and $R(h, y_2)$, where $h \neq 0$, find the value of the integral $\frac{1}{h} \int_{-h}^h f(x) dx$.

- A. $\frac{y_0 + y_1 + y_2}{3}$ B. $\frac{y_0 + 2y_1 + y_2}{3}$ C. $\frac{y_0 + 4y_1 + y_2}{3}$ D. $\frac{y_0 + 4y_1 + y_2}{6}$ E. ***

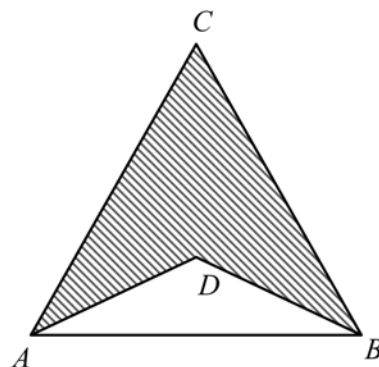
16. 有多少个实数 x 满足方程式 $e^x = 2025x$?

How many real numbers x satisfy the equation $e^x = 2025x$?

- A. 0 B. 1 C. 2 D. 3 E. ***

17. 右图中， $\triangle ABC$ 与 $\triangle ABD$ 是等腰三角形。若 $AB = 12$ ， $\angle CAB = \alpha$ ， $\angle DAB = \beta$ ，阴影部分的面积为 144，求 $\tan \alpha - \tan \beta$ 的值。

In the figure on the right, $\triangle ABC$ and $\triangle ABD$ are isosceles triangles. Suppose $AB = 12$, $\angle CAB = \alpha$, $\angle DAB = \beta$, and the area of the shaded region is 144, find the value of $\tan \alpha - \tan \beta$.



- A. $\frac{1}{2}$ B. 1 C. 2
D. 4 E. ***

18. 当 $n \geq 1$ 时, 定义 $S_n = \sum_{k=1}^n (2k-1)^7 = 1^7 + 3^7 + \cdots + (2n-1)^7$. 求极限 $\lim_{n \rightarrow \infty} \frac{S_n}{n^8}$.

For $n \geq 1$, define $S_n = \sum_{k=1}^n (2k-1)^7 = 1^7 + 3^7 + \cdots + (2n-1)^7$. Find the limit $\lim_{n \rightarrow \infty} \frac{S_n}{n^8}$.

- A. 2 B. 16 C. 32 D. 256 E. ***

19. 已知 $a - b = 1$, $b - c = 2$, $c - d = 3$, 求

$$a^2 + b^2 + c^2 + d^2 - ab - bc - cd - da$$

的值。

Given that $a - b = 1$, $b - c = 2$, $c - d = 3$, find the value of

$$a^2 + b^2 + c^2 + d^2 - ab - bc - cd - da.$$

- A. 15 B. 25 C. 30 D. 50 E. ***

20. 设 $p(x) = c_6x^6 + c_5x^5 + c_4x^4 + c_3x^3 + c_2x^2 + c_1x + c_0$ 是一六次多项式, 其系数 $c_0, c_1, c_2, c_3, c_4, c_5, c_6$ 均为实数。若对于所有的实数 x , $p(x)$ 都大于或等于 0, 则最多有几个实数 x 使得 $p(x) = 0$?

Let $p(x) = c_6x^6 + c_5x^5 + c_4x^4 + c_3x^3 + c_2x^2 + c_1x + c_0$ be a polynomial of degree six with real coefficients $c_0, c_1, c_2, c_3, c_4, c_5, c_6$. If for all real numbers x , $p(x)$ is greater than or equal to 0, then at most how many real numbers x can there be such that $p(x) = 0$?

- A. 0 B. 3 C. 4 D. 6 E. ***

21. 设 \mathbb{Z}^+ 为正整数集合。定义函数 $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ 使得 $f(n)$ 为正整数 n 的各位数字之和。例如， $f(1234) = 1 + 2 + 3 + 4 = 10$ 。求

$$\sum_{n=1}^{99} f(n) = f(1) + f(2) + \cdots + f(99)$$

的值。

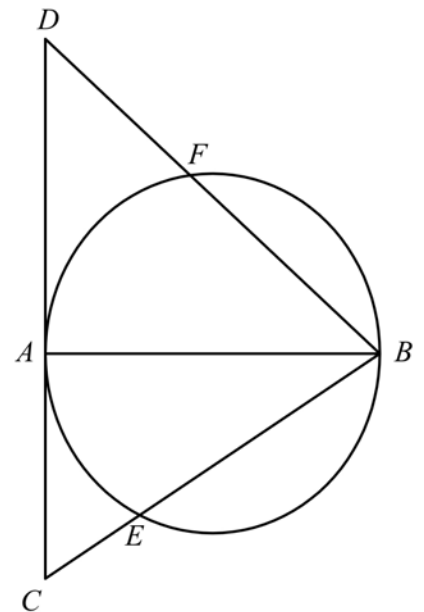
Let \mathbb{Z}^+ be the set of positive integers. The function $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ is defined so that $f(n)$ is the sum of the digits of n . For example, $f(1234) = 1 + 2 + 3 + 4 = 10$. Find the value of

$$\sum_{n=1}^{99} f(n) = f(1) + f(2) + \cdots + f(99).$$

- A. 900 B. 855 C. 810 D. 800 E. ***

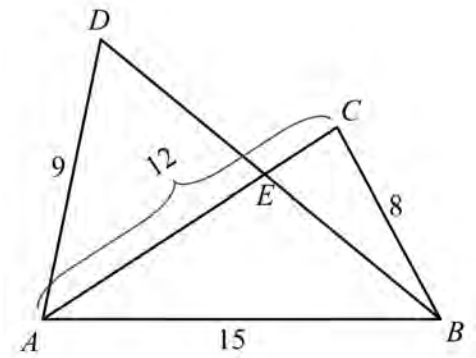
22. 右图中， AB 是圆的直径， CAD 是圆的切线。直线 BC 及 BD 分别与圆相交于点 E 及 F 。已知 $CE:CB = 4:13$ 及 $DF:DB = 1:2$ ，求 $\tan \angle CBD$ 。

In the figure on the right, AB is a diameter of the circle and CAD is a tangent to the circle. Lines BC and BD intersect the circle at points E and F respectively. Given that $CE:CB = 4:13$ and $DF:DB = 1:2$, find $\tan \angle CBD$.



- A. 2 B. $\frac{5}{2}$ C. 4
D. 5 E. ***

23. 右图的 $\triangle ABC$ 与 $\triangle ABD$ 中, AC 与 BD 边相交于点 E 。
已知 $AB = 15$, $BC = 8$, $AC = 12$, $AD = 9$, 且 $\angle BAD + \angle ACB = 180^\circ$, 求 $\frac{DE}{BE}$ 的值。



In $\triangle ABC$ and $\triangle ABD$ shown on the right, AC and BD intersect at point E . Given that $AB = 15$, $BC = 8$, $AC = 12$, $AD = 9$, and $\angle BAD + \angle ACB = 180^\circ$, find the value of $\frac{DE}{BE}$.

- A. $\frac{9}{10}$ B. $\frac{5}{4}$ C. $\frac{4}{5}$
D. $\frac{3}{5}$ E. ***

24. 已知 a_1, a_2, a_3, a_4, a_5 是正数, 且 $a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 = 5$. 求

$$\frac{1}{a_1} + \frac{8}{a_2} + \frac{27}{a_3} + \frac{64}{a_4} + \frac{125}{a_5}$$

的最小可能值。

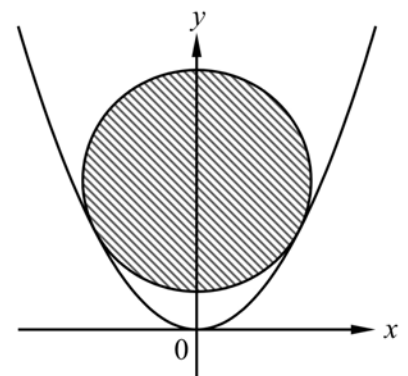
Given that a_1, a_2, a_3, a_4, a_5 are positive numbers and $a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 = 5$. Find the smallest possible value of

$$\frac{1}{a_1} + \frac{8}{a_2} + \frac{27}{a_3} + \frac{64}{a_4} + \frac{125}{a_5}.$$

- A. 225 B. 275 C. 605 D. 979 E. ***

25. 右图中, 圆与抛物线 $x^2 = 2y$ 相切。若圆心的坐标为 $(0, 7)$, 求圆的面积。

In the figure on the right, a circle is tangent to the parabola $x^2 = 2y$. Given that the circle is centered at $(0, 7)$, find the area of the circle.



- A. 7π B. 13π C. 14π
D. 28π E. ***

26. 求使得 5^k 可以整除 $2024! \times 2025!$ 的最大正整数 k 。

Find the largest positive integer k such that 5^k divides $2024! \times 2025!$.

- A. 909 B. 910 C. 1000 D. 1008 E. ***

27. 设函数 $f_1 : \mathbb{R} \rightarrow \mathbb{R}$ 的定义为 $f_1(x) = \frac{3x+8}{x+1}$ 。当 $n \geq 2$ 时, 函数 $f_n : \mathbb{R} \rightarrow \mathbb{R}$ 的定义为 $f_n(x) = (f_{n-1} \circ f_1)(x)$ 。求 $f_{2025}(-2)$ 的值。

Let $f_1 : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f_1(x) = \frac{3x+8}{x+1}$. For $n \geq 2$, the function $f_n : \mathbb{R} \rightarrow \mathbb{R}$ is defined recursively by $f_n(x) = (f_{n-1} \circ f_1)(x)$. Find the value of $f_{2025}(-2)$.

- A. -2 B. 0 C. 2 D. 2025 E. ***

28. 求方程式 $x = 3^{4+\log_x 9}$ 的所有实根的乘积。

Find the product of all real roots of the equation $x = 3^{4+\log_x 9}$.

- A. $\frac{1}{81}$ B. $\frac{1}{9}$ C. 9 D. 81 E. ***

29. 设 $P = \{1, 2, 3, 4, 5\}$, $Q = \{a, b, c\}$ 。任选一个由集合 P 到集合 Q 的函数 $f : P \rightarrow Q$, 求它是映成函数的概率。

Let $P = \{1, 2, 3, 4, 5\}$ and $Q = \{a, b, c\}$. A function $f : P \rightarrow Q$ is chosen randomly from set P to set Q , find the probability that it is a surjective (onto) function.

- A. $\frac{49}{81}$ B. $\frac{50}{81}$ C. $\frac{148}{243}$ D. $\frac{149}{243}$ E. ***

30. 设 p_n 为从所有 n 位正整数中随机选择一个数使得任意两个数字都不相等的概率。求满足 $p_n < 0.5$ 的最小正整数 n 。

Let p_n be the probability of choosing an n -digit positive integer randomly such that no two digits are equal. Find the smallest positive integer n satisfying $p_n < 0.5$.

- A. 3 B. 4 C. 5 D. 6 E. ***

31. 若 x 及 y 是实数且 $x^2 + y^2 + 10x - 24y + 133 = 0$, 求 $x^2 + y^2$ 的最小可能值。

If x and y are real numbers such that $x^2 + y^2 + 10x - 24y + 133 = 0$, find the smallest possible value of $x^2 + y^2$.

- A. 6 B. 7 C. 36 D. 49 E. ***

32. 求满足方程式 $m^2 + 5n^2 = 2025$ 的正整数对 (m, n) 的个数。

Find the number of pairs of positive integers (m, n) satisfying the equation $m^2 + 5n^2 = 2025$.

- A. 2 B. 8 C. 12 D. 20 E. ***

33. 求满足不等式

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{2025}} \leq m\sqrt{2025}$$

的最小正整数 m 。

Find the smallest positive integer m satisfying the inequality

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{2025}} \leq m\sqrt{2025}.$$

- A. 1 B. 2 C. 45 D. 2024 E. ***

34. 右图的 $\triangle ABC$ 中, AD, BE, CF 分别为 $\angle BAC, \angle ABC, \angle ACB$ 的角平分线。若 $AB = 4, AC = 5, BC = 6$, 求

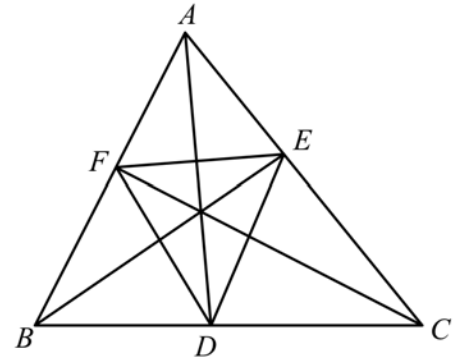
$$\frac{\triangle DEF \text{ 的面积}}{\triangle ABC \text{ 的面积}}$$

的值。

In $\triangle ABC$ shown on the right, AD, BE, CF bisect $\angle BAC, \angle ABC, \angle ACB$ respectively. If $AB = 4, AC = 5$ and $BC = 6$, find the value of

$$\frac{\text{area of } \triangle DEF}{\text{area of } \triangle ABC}$$

- A. $\frac{8}{33}$ B. $\frac{1}{4}$ C. $\frac{5}{18}$
 D. $\frac{1}{3}$ E. ***



35. 整数 12 的正因子为 1, 2, 3, 4, 6, 12, 因此我们说 12 恰有 6 个正的因子。从 1 到 200 的整数中, 有多少个整数恰有 6 个正的因子?

The positive factors of integer 12 are 1, 2, 3, 4, 6, 12. Thus, we say that 12 has exactly 6 positive factors. Among all integers from 1 to 200, how many of them have exactly 6 positive factors?

- A. 25 B. 26 C. 27 D. 28 E. ***

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